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A Method for Measuring the Bending and Torsional Stiffness Distributions of Alpine Skis

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Abstract

A novel non-destructive method for quickly, accurately and simultaneously measuring the bending and torsional stiffness distributions of an alpine ski is presented. This method, named SMAD (Stiffness Measurement through Angular Deformations), is based on measuring the angular deformations resulting from a known combined bending and torsion load. The method's accuracy and repeatability is investigated and are on average under 2% and 3%, respectively. The coupling in the measurement of the bending and torsional deformations during combined loading due to ski misalignment in the test machine is investigated. The measured torsional deformation was found to be independent of the bending load. The measured bending deformation was found to be dependent on the torsional load but this effect could be limited by careful alignment of the ski in the test machine.

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Keywords: Stiffness; Rigidity, Bending, Flexural, Torsional, Measurement

1. Introduction

An accurate method for the measurement of the bending and torsional stiffness distributions of alpine skis is highly desirable, as these mechanical properties play a major role in determining how a ski will perform [1]. Such a method can find many applications in the areas of research and development, quality control, product reviews and online retail. Over the years, numerous methods have been developed to measure an alpine ski's bending and torsional stiffnesses. ISO Standard 5902 [2] describes a test procedure to determine average bending spring constants for the forebody, afterbody and center of an alpine ski, as well as torsional spring constants for the fore and afterbody. In this method, one end of the section of interest is clamped while the deflection resulting from a known bending or torsional load applied at the free end is measured. Methods to obtain the distribution of bending stiffness (i.e., $EI(x)$, the ratio of an applied bending moment and the beam's resulting curvature) and torsional stiffness (i.e., $GJ(x)$, the ratio of an applied torque and the resulting twist angle rate-of-change) of structural beams along their length (e.g., alpine skis, golf shafts, hockey sticks) also exist and are based on the measurement of the beam deformation profile under a known load. Methods for applying a load to the ski include 3-point bending tests [3,4,5], cantilever/end-load bending tests [6], as well as cantilever/end-torque torsion tests [3,4,5]. Variations on such methods have also been proposed, such as multiple tests on short segments, where each test location is moved along the length of the beam [6]. Methods for measuring the deformation profile include measuring the vertical deflection through the use of a laser transducer [3], an LVDT transducer contacting the surface of the ski [4] as well as an infrared tracking system coupled with reflective markers mounted to the ski [4]. Another method for obtaining the deformation profile consists in estimating the curvature of the beam using a digital radius gauge [5]. Due to the variable geometry of skis and the large number of new skis to test annually, it is desirable to have a method of characterization that is both accurate and faster than existing methods.

This paper presents a novel non-destructive method for rapidly obtaining high-resolution bending and torsional stiffness distributions of an alpine ski with few manipulations. The SMAD (Stiffness Measurement through Angular Deformations) method,

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described in Section 2, is particularly suitable for the acquisition of large datasets and is based on the measurement of angular deformations created by a combined bending and torsional load. Section 3 evaluates the performance of the SMAD method. More specifically, its accuracy is validated by comparing the measurements taken on a prismatic, homogeneous beam with the average bending and torsional stiffnesses obtained from a 3-point bending test and a cantilever/end-torque torsion test. Multiple measurements are also repeated on a single alpine ski in order to assess the method's repeatability and sensitivity to operator errors. Finally, the coupling between the measured bending and torsion deformations is evaluated by comparing the stiffness distributions obtained by applying combined bending and torsion loads of different relative magnitudes.

2. Method

The SMAD method is based on the measurement of angular deflections. When a combined load is applied on an alpine ski, both bending and torsional stiffness profiles can be calculated from the simultaneous measurement of the bending and torsional angular deformation profiles. This section first describes the experimental setup used to bend and twist the ski as well as the measuring instruments. Then, the calculations required to estimate the stiffnesses from the angular deformation profiles are explained.

2.1. Experimental Setup

The apparatus used to apply a load to the ski and measure the resultant angular deformations is shown in Figure 1.

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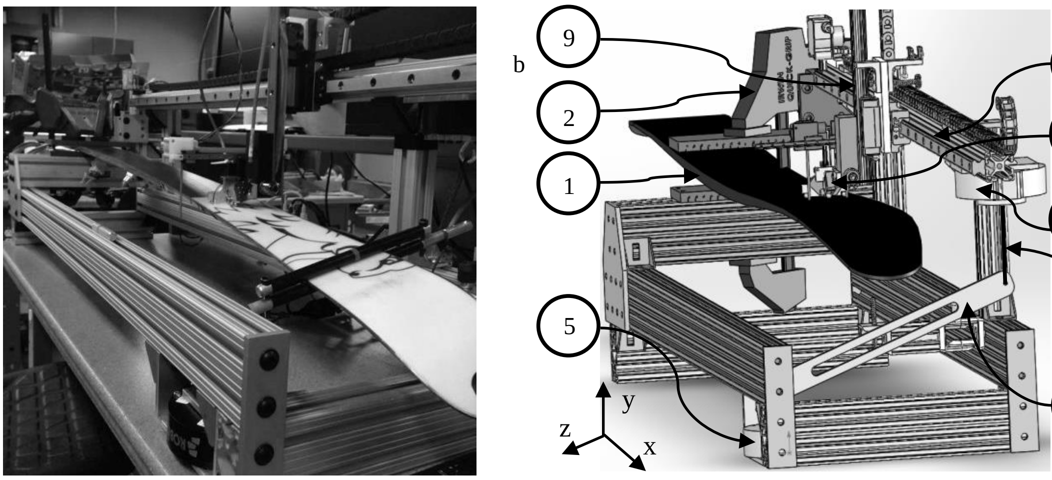

Fig. 1. Experimental setup illustrated by a a) photo of the physical apparatus and b) a CAD drawing

The ski 1 is fixed in the clamp 2 upside-down near the boot area. A combined bending and torsion load is applied near the free end (i.e., tail or tip) through the fixture 3 consisting of two cylinders. Each end of the fixture is connected through ropes 4 to 3-axis force transducers 5 and 6 to measure the load and calculate the bending and torsional moment at all points along the ski. This setup creates a triangular distribution of bending moment that roughly matches the bending stiffness profile of a half-ski (i.e., the moment is greatest near the boot area, where the stiffness is the greatest, and reaches zero at the tip).

The measurement device 7, shown in Figure 2, consists of three spherical followers sliding along the surface of the ski base due to the sliders 8 and 9. The two rear followers 10 are mounted to the body 11 that is free to rotate about an axis parallel to the ski's torsional deformation, $\phi(x)$. This body, as well as the front follower 12, is mounted to the second body 13 free to rotate about the Z-axis and corresponding to the ski's bending deformation, $\theta(x)$. These two angular deformation distributions are measured using optical encoders 14 with a resolution of 0.009°.

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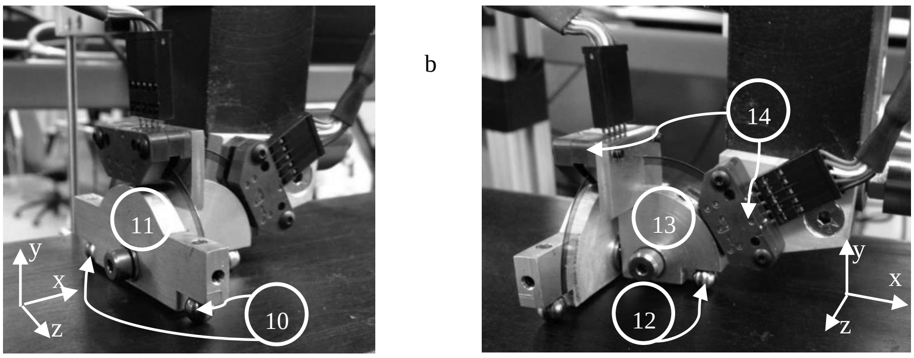

Fig. 2. Curvature measurement apparatus from a) back view and b) front view.

To obtain the stiffness distributions on the full length of the ski, the ski is divided in two overlapping sections, front and back, which are measured in separate tests. Testing only half the ski at a time in a clamped/free configuration facilitates the application of a combined bending and torsional load to the ski while still keeping the ideal triangular distribution of the bending moment. If the whole length of the ski were tested at once, a 3-point flexural test would be necessary to obtain the same triangular bending moment distribution. However, such a configuration would greatly complicate the simultaneous application of a torsional load.

For each half, the deformation is calculated by subtracting the measured unloaded shape (i.e., camber) from the deformed state under the applied load. Both tests results are combined using a reference marker placed in the overlapping region, while the measurements in this region are averaged together.

2.2. Stiffness calculations

The ski's bending stiffness (EI) is calculated with equation 1.

$$EI(x) = \frac{M_z(x)}{\kappa(x)} \tag{1}$$

Where M_z is the bending moment applied to the ski and κ is the resultant curvature. The curvature, which is the derivative of the bending angle (θ) with respect to the arc length s, is approximated with the derivative with respect to the horizontal distance x:

$$\kappa(x) = \frac{d\theta}{ds} \approx \frac{d\theta}{dx} \tag{2}$$

This approximation leads to loss of accuracy of less than 1% for a typical ski. Similarly, the torsional stiffness (GJ) was calculated as the ratio of the applied torsion moment M_x to the resultant rate of change of the torsion angle with respect to x:

$$GJ(x) = \frac{M_x(x)}{d\phi / dx} \tag{3}$$

The derivatives $d\theta/dx$ and $d\phi/dx$ are themselves calculated using a centered finite differences ratio over a spatial step of 10 cm ($\Delta x = 0.05\text{m}$) such that:

$$\frac{d\phi}{dx} = \frac{\phi_{x+\Delta x} - \phi_{x-\Delta x}}{2\Delta x}, \qquad \frac{d\phi}{dx} = \frac{\phi_{x+\Delta x} - \phi_{x-\Delta x}}{2\Delta x} \tag{4}$$

A smaller spatial step size averages the stiffness measurement over a shorter interval. This is desirable to obtain a faithful reproduction of the actual stiffness distribution, especially near abrupt geometry or laminate changes. However, the step size is limited by the angular measurements resolution and/or noise. A step size of 10 cm was chosen as it provides a balance between these two conflicting requirements.

The bending and torsion moment distributions were calculated from the force transducer readings ($\vec{F}_{sensor}$, ± 0.5 N) and their position ($\vec{r}_{sensor}$, ± 1 mm) with the following equation:

$$\overrightarrow{M}(x) = \sum\left(\left(\vec{r}_{sensor} - \vec{r}(x)\right) \times \vec{F}_{sensor}\right) \tag{5}$$

Where the position of the neutral axis, $\vec{r}(x)$, is estimated from the position of the measurement device (x, ± 1 mm) and from integrating the bending angle and setting the initial angle at the clamp to zero according to the following equation:

$$\vec{r}_x = \begin{bmatrix} x \\ \int_0^x \theta dx \\ 0 \end{bmatrix} \tag{6}$$

3. Results and discussion

Three main tests were carried out in order to assess the above method's accuracy, its repeatability, as well as to investigate the presence of any coupling in the measurement of the bending and torsional deformations.

3.1. Method accuracy

In order to validate the accuracy of the method, a 25.6 mm ± 0.1 mm thick, 77.7 mm ± 0.5 mm wide prismatic beam was manufactured from a polyoxymethylene polymer (Nytef Plastics Unital C, $E = 2.52$ GPa). Such a beam has a similar bending stiffness as existing alpine skis: the nominal bending stiffness *EI* is 274.1 Nm². The shear modulus for the material used, necessary for estimating the nominal torsional stiffness, was not available from the manufacturer and literature values for various polyoxymethylene polymers can vary significantly. As such, the nominal torsional stiffness was not calculated. This beam's bending and torsional stiffness distributions were measured using the method described in the previous section. Both test results are illustrated in Figure 3.

For comparison, the average bending stiffness of the beam was evaluated with a 3-point bending test. The beam was simply supported at both ends on rollers spaced 1095 mm ± 2 mm apart (*L* = 1095 mm) and a load *F* of 131.9 N ± 0.05 N was applied at midpoint. The resulting central deflection v was measured with a digital caliper (13.2 ±0.1 mm) to calculate the average bending stiffness with equation 7 [7], which resulted in an average bending stiffness of 272.6 Nm² ± 3.7 Nm².

$$EI = (FL^3)/48v \tag{7}$$

The average torsional stiffness was evaluated by clamping the beam at one end and applying a torque at the other extremity. The test was carried out on a 788 mm ± 1 mm long segment (*L* = 788 mm) with an applied torque *T* of 30.0 Nm ± 0.3 Nm. The resulting angular deflection at the free end was then measured (3.58° ± 0.03°) in order to calculate the average torsional stiffness with equation 8, resulting in a value of 377.7Nm² ± 6.8 Nm².

$$GJ = TL/\Delta\varphi \tag{8}$$

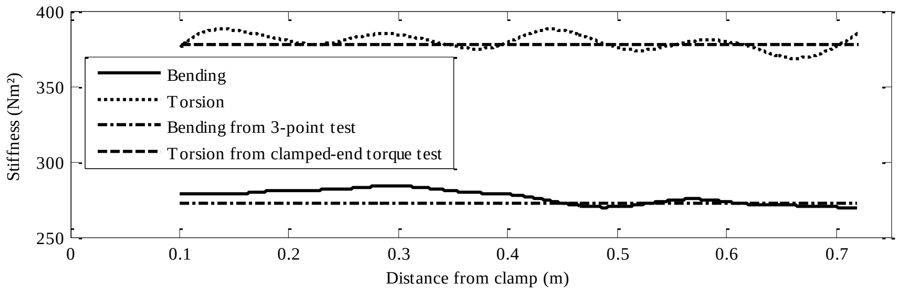

Fig. 3. Bending and torsional stiffnesses of a prismatic polymer beam with two methods

The previously described method's accuracy was evaluated by comparing the stiffness distributions to the average-test stiffnesses. Figure 4 shows the difference between these measurements in both bending and torsion. The maximum absolute difference between the measured distributions and the measured average stiffnesses are 4.1% in bending and 2.7% in torsion, while the average absolute differences are 1.8% in bending and 1.1% in torsion. These discrepancies are slightly larger than those expected because of the beam's geometry variation, which could account for stiffness variations of up to 1.8%, but could realistically be explained by slight variations in the material's mechanical properties.

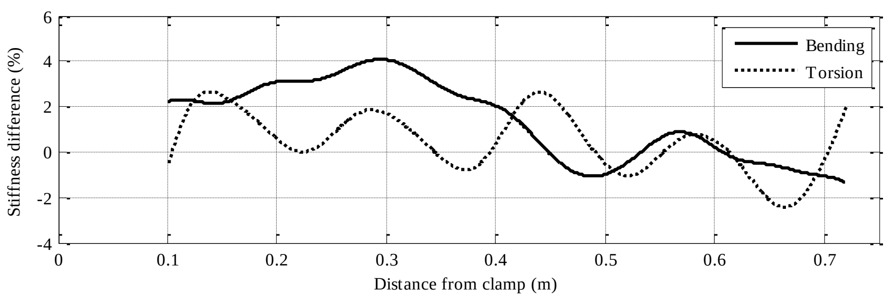

Fig. 4. Difference between local and average stiffnesses in both bending and torsion

3.2. Method repeatability

The method's repeatability was assessed by conducting five tests on an actual ski, a Dynafit Se7en Summit 2013 (178 cm), to measure stiffness curves for which the apparatus has been designed. On average, each test required less than 6 minutes to perform. Figures 5 and 6 show the average stiffnesses of the 5 tests and the standard deviations in both bending and torsion.

Several factors influence the method's repeatability. Surface roughness and imperfections are present on all ski bases. If the ski is moved slightly between tests, the followers of the reading head will not trace the exact same line before and after the application of the load and these imperfections will not cancel each other out. To reduce this effect, it is possible to filter the measurements. However, filtering assumes a certain data structure and might hide abrupt changes in stiffness.

Also, the choice of the clamped-free loading method induces variable uncertainty depending of the position along the length of the ski. The bending moment is greatest in the center of the ski, near the clamp, and nears zero at the tips, near the load application points. This small bending moment at the tips induces a large relative error as $\left(\vec{r}_{sensor} - \vec{r}\left(x\right)\right)$ tends towards zero, which explains why the bending standard deviation increases further away from the center of the ski. In torsion, as the torque varies little along the length of the ski, the greater stiffness at the center of the ski leads to smaller angular variations and therefore larger relative uncertainties due to the sensor's finite resolution. Even so, the standard deviations are mostly under 5% with an average of 2.63% in bending and 2.80% in torsion.

The two peaks in the torsional error standard deviation, although unusual, cannot be explained by gouges or dents in the ski base (which was smooth visually) and appear to be random. One possible explanation is the small sample size; with only five tests, a random large error spike in a single test is enough to have a noticeable effect on the standard deviation. It is also worth noting these peaks occur in the central area in which the uncertainty is expected to be higher.

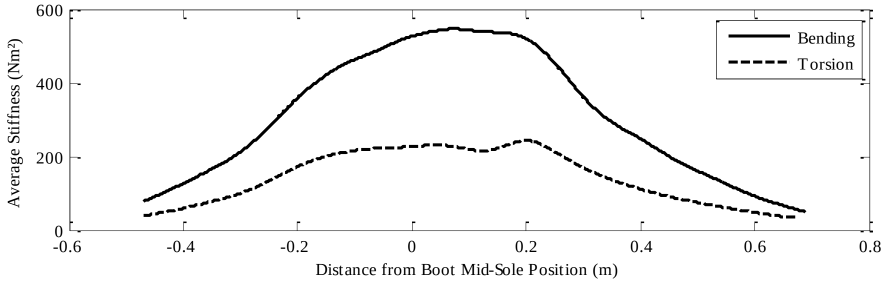

Fig. 5. Average stiffnesses of 5 consecutive tests on a Dynafit Se7en Summit ski

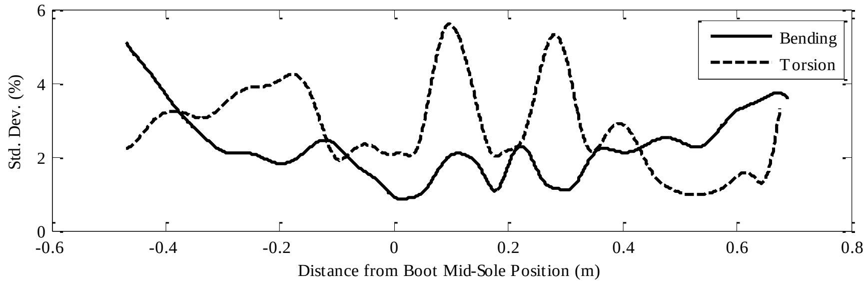

Fig. 6. Standard deviation of 5 consecutive tests.

3.3. Coupling of bending and torsional deformations

To accelerate data acquisition, the SMAD method can simultaneously measure the bending and torsional stiffnesses. To obtain accurate results, this method requires that there be no or negligible coupling in the measurement of the bending and torsional deformations. While it is possible to design a composite beam with bending-torsion coupling, alpine skis generally have no such coupling due to the symmetry of geometry and materials layup about the sagittal (XY) plane. However, if there is a misalignment between the ski and the test bench x-axis (e.g., due to a rotation about the y-axis of the ski in the clamp), a coupling may occur in that the bending angular sensor may measure a portion of the torsional deformation, and vice-versa.

To quantify this effect, the Dynafit Se7en Summit ski was measured three times using the same setup and method, but with different applied loads. For the first test case, the ski was loaded mostly in bending ($M_x = 9$ Nm and $\max M_z = 60$ Nm). The second test case was conducted with the ski mostly loaded in torsion ($M_x = 30$ Nm and $\max M_z = 20$ Nm). The third test case was conducted with a combined loading ($M_x = 24$ Nm and $\max M_z = 45$ Nm), as typically applied when testing skis with this instrument. Figure 7 shows the bending and torsional stiffnesses obtained from these tests. Figure 8 shows the variation between the stiffnesses obtained from the three test cases.

The difference between the torsional stiffness distributions as measured in test cases 2 and 3 averages 2.55%, with a maximum difference of 7.5%. This is largely consistent with the repeatability error presented in the previous section, which indicates that the torsional deformation distribution appears to be independent of the bending load applied to the ski.

In bending, the average difference in the calculated stiffness distribution between test cases 1 and 3 is 3.43%, with a maximum difference of 10.78%. The difference is larger at the tips of the ski and smaller in the center, which is also consistent with the trends observed in the repeatability test. However, the standard deviation at the tips from the repeatability tests did not exceed 5%, whereas the difference in this test is over 10%. This larger discrepancy is explained by ski misalignment during installation in the clamp. This misalignment causes the read head measurement line to be at an angle relative to the ski's true centerline, causing the read head to measure a component of the torsional deformation in addition to the bending deformation. Therefore, the measured bending stiffness is not completely decoupled from the torsional load applied, and varies with the amplitude of the ski misalignment in the clamp. This error was deemed acceptable, as the difference is under 5% for much of the length of the ski, and the larger discrepancies occur at the tips of the ski, where the stiffness is of less interest. However, the test machine could easily be modified to allow for more precise alignment should the need for more precise results arise (in its current embodiment, misalignment can be up to 0.6°).

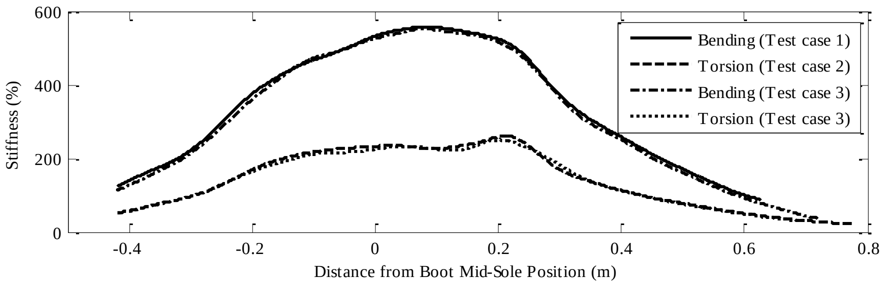

Fig. 7. Bending and torsional stiffnesses for three different test cases

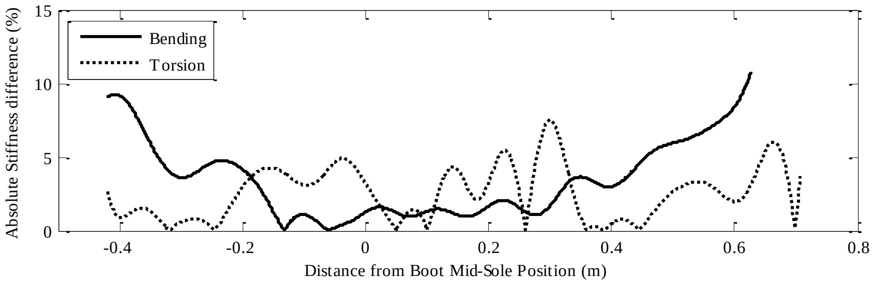

Fig. 8. Absolute stiffness differences between test cases 1 and 3 in bending and 2 and 3 in torsion

4. Conclusion

The method developed to simultaneously measure the bending and torsional stiffness distributions of an alpine ski, based on the measurement of angular deformations, was shown to be accurate and repeatable. The average stiffness error on a control beam did not exceed 1.8 % in bending and 1.1 % in torsion. The repeatability of the test method was evaluated with 5 consecutive tests on an actual ski and was shown to be repeatable within at most 6%, with the average standard deviation not exceeding 3%.

The coupling of the measured bending and torsional deformations during simultaneous measurement on an alpine ski was investigated, as any misalignment between the ski and the test machine will yield such a coupling. The torsional stiffness results are not affected by this misalignment. However, in bending, it was determined that the method is sensitive to the ski's alignment in the test machine. In its present embodiment, the test machine yields an error mostly under 5% near the center of the ski, and mostly under 10% near the tips.

The presented method, which can be used to measure a ski's bending and torsional stiffness distributions in approximately 6 minutes per ski, has the potential to rapidly yield large sets of data on many alpine skis. These datasets can be of significant use in research & development, quality control, product reviews and online retail.

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